COMPARISON BETWEEN THEORETICAL AND NUMERICAL SOLUTIONS

FOR CENTER, SINGLE EDGE AND DOUBLE EDGE CRACKED FINITE

PLATE SUBJECTED TO TENSION STRESS

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ABSTRACT

The stress intensity factor is one of the most important concepts in fracture mechanics because of stresses near the crack tip increase in proportion to it. Therefore, it must be calculated accurately to obtain the fracture damage. This paper deals with determination of the stress intensity factor mode I (KI) for Centre Craked Tension (CCT), Single Edge Notch Tension (SENT) and Double Edge Notch Tension (DENT) finite platesunder Linear Elastic Fracture Mechanics (LEFM) and plane strain assumptions. To investigate the differences between the theoretical and numerical solutions, a comparison is made between the stress intensity factors calculated using a large number of standard equations with the others calculated using finite element software ANSYS R14.5.Stress intensity factors are evaluated with variation of crack length to plate width ratio (a/b), tensile stresses (σ_t) and plate length to plate width (h/b). The analysis shows that the numerical solution is more suitable than the theoretical solution due to the results that the theoretical equations are limited in a range of parameters values, some parameters such as length plate is not considered in it and also the difficulty to determine the accurate stress intensity factors for cracks in complex structures.

KEYWORDS: Stress Intensity Factor, Centre Craked Tension (Cct), Single Edge Notch Tension (Sent), Double Edge Notch Tension (Dent), Ansys R14.5

INTRODUCTION

The prediction of the mechanical behavior of a structure when it is subjected to an external load such as tension represents the basic problem in mechanics. In the design, our aim is to predict the deformation and stresses that happen in the material to avoid catastrophic failures occurred. Stresses are concentrated and reached to a large values at the geometry discontinuities as holes, corners, notches, cracks etc .

The fracture mechanics theory can be used to analyze structures and machine components with cracks and to obtain an efficient design. The basic principles of fracture mechanics developed from the studies of Inglis, Griffith and Trans are based on the concepts of linear elasticity. Westergaard derived the general linear elastic solution for the stress field around a crack tip using complex stress functions, Bhagat and et at. [1].Irwin was one of the first to study the behavior of cracks. He introduced three different loading modes (Figure 1), which are still used today:Mode I = opening mode, Mode II = sliding mode and Mode III =tearing mode, Schreurs[2].

Huang and Kardomateas [3] formulated an approach based on the continuous dislocation technique to obtain the stress intensity factors for mode I and II in a fully anisotropic infinite strip with a central crack. Chin [4] used the finite element software ANSYS and the crack growth software FRANC3D to investigate how a crack propagates and grows in a

typical Ti-6Al-4V aerospace bracketby computing the stresses and the stress-intensity factor. A specific bracket design was selected and a corner crack was investigated. The effect of crack inclination angle on stress intensity factors for two parallel and non parallel cracks using ANYSIS software was studed by Bhagat and et at. [1].Ergun and etat. [5] used the Finite Element Method (FEM) to analyze the behavior of repaired cracks in 2024-T3 aluminum with bonded composite .Stress intensity factors mode I were calculated based on displacement correlation technique.Ali and et at. [6]analyzed the stress intensity factor in various edge cracks along the length of a finite plate which was under a uniformtension using FEM. Al-Ansari [7] calculated a stress intensity factor modeI of unidirectional composite center cracked plate withthe effect of volume fraction of unidirectional fiber using FEM.

To predict the behavior of a crack, it is essential to know its location, geometry and dimensions. In this paper attention is given to calculate the stress intensity factor for center, single edge notch and doubleedge notch crakedfinite plates. Throughout thispaper, it is assumed that dimensionsthe material behavior is linear elastic and isotropic.

MATERIALS AND METHODS

Finite-width plate specimenswith Centre Cracked Tension (CCT), Single Edge Notch Tension (SENT) and Double Edge Notch Tension (DENT) subjected to uniform uni-axial tension stresses in the Y- direction as shown in Figure 2 are studied to determine the stress intensity factors mode I (KI) using numerical and theoretical solutions under the assumptions of Linear Elastic Fracture Mechanics (LEFM) and plane strain.

Specimens Material

The material of plates is carbon steel with modulus of elasticity = 202 e-3 MN/m^2 and poison's ratio= 0.292.

Theoretical Solution

Previously, a large number of equations were formulated to calculate approximately the stress intensity factors. In this paper, five different theoretical equationsfrom different references are selected to calculate the stress intensity factorsforeach type of the three platespecimens as shown in the Table1.

Numerical Solution

Stress intensity factors are calculated numerically using finite element software ANSYS R14.5with PLANE183 element as a discretization element. As a reason of symmetry, half model is used in SENT case and quarter models are used in case of CCT and DENT as shown in Figure 2.CCT, SENT and DENT ANSYS models are shown in Figure 3 with the mesh, elements, nodes and boundary conditions.

PLANE183 Description

PLANE183 = element type is defined by 8 nodes (I, J, K, L, M, N, O, P when quadrilateral element) or 6 nodes (I, J, K, L, M, N when triangle element) having two degrees of freedom (UX, UY) at each node (translations in the nodal X and Y directions). It has quadratic displacement behavior and is well suited to modeling irregular meshes.Element loads are described in nodal loading. Pressures may be input as surface loads on the element faces. Positive pressures act into the element and temperatures may be input as element body loads at the nodes, ANSYS help [11]. The geometry, node locations, and the coordinate system for this element are shown in Figure 4. PLANE183 is used in this paper as a discretization element with quadrilateral shape, plane strain behavior and pure displacementformulation.

The Studied Cases

To illustrate the difference between theoretical and numerical solutions for CCT, SENTand DENTthe stress intensity factors for many cases are studied theoretically using a large number of equations and numericallyusing finite element software ANSYS R14.5. The results of all studied cases are reported in Table 2.

RESULTS AND DISCUSSIONS

KI values are theoretically calculated using different equations and numerically using ANSYS R14.5 for three differenttypes of cracked plate specimens (CCT, SENT and DENT) with different casesreportedin Table 1.

The Numerical Compression

Case Study I

Figure 5-7 explain the numerical and theoretical variations of stress intensity factors KI with different values of a/b ratio as shown in Table 2. From these figures, it can be seen that increasing the ratio of a/b leads to increasing the value of $K_I$in a high level for all the selected values. These figures also show afitness between theoretical and numerical solutions at a/b=0.3 for SENT and at a/b<0.25 for CCT and DENT. A small difference occurs between the two solutions at a/b>0.25 for CCT and at a/b><0.3 for SENTwhile a sensitive difference occurs at a/b>0.25 for DENT.

Case Study II

Figure 8-10 show the variation of KI with different values of tension stresses (shown in Table 2). It can be seen from these figures that there are fitness between numerical and theoretical solutions for all plate specimen types (CCT, SENT and DENT) except small difference, (7) and (12)for SENT and DENT, respectively.

Case Study III

Figure 11-13 illustrate the theoretical and numerical variations of KI with different values of h/b ratio as shown in Table 2.From all these figures, we shown that a large difference between numerical and theoretical solution at h/b<1 but this difference will vanish at h/b>1 except of (7) and (12) for SENT and DENT, respectively.

The variation of numerically computed KI for the three of different cracked plate specimens (CCT, SENT and DENT) in I, II and III cases are shown in Figure14-16, respectively. In all mentioned figures, it is found that the KI values in CCT are less than that of SENT and DENT. From Figure 14 and Figure 15, it is clear that the values of KI for SENT are greater than that from DENT, while from Figure 16 we shown that the values of KI equal at h/b <0.5 and after then when h/b>0.5 the values of KI for DENT will be greater than that for SENT.

Furthermore, Figures 17a, b, c, d, e, f graphically illustrated Von-Mises stresses countor plots with deformed shape for the cracked plate specimens for different states. Figures 17a, c, e show the variation of $\sigma_{\text{Von-mises}}$ for CCT, SENT and DENT, respectively when b=0.2m,h=0.2m, a=0.03m and σ_t=200Mpa. The variation of $\sigma_{\text{Von-mises}}$ for CCT, SENT and DENT, respectively when b=0.2m, h=0.2m, a=0.08m and σ_t=100Mpa are shown in Figures 17b, d, f.

CONCLUSIONS

- KI values are calculated from the theoretical equations for different cases are approximately same in all cases except the values calculated from (12) and (15) for the case of DENT, (2) in case of CCT and (7) in case of SENT.

- Numerical solution results coincide with the theoretical solutions for many cases except when h/b<1. At this point, KI is not suitable to calculate using theoretical approach because it doesn't take this parameter in consideration.

- Numerical solution is more suitable to calculate KI due to all theoretical equations are limited for some cases and for regular geometry while the numerical solution is used for all cases and with regular and irregular shapes.

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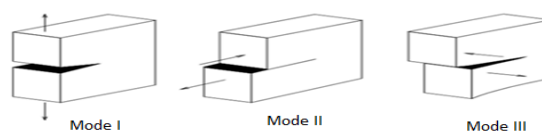10. V. E. Saouma. Lecture Notes in: FRACTURE MECHANICS-Chapter4. http://civil.colorado.edu/~saouma/Lecture-Notes/lecfrac.pdf.

11. ANSYS help.

APPENDICES

Figure 1: Three Standard Loading Modes of a Crack, Schreurs [2]

Comparison between theoretical and Numerical Solutions for Center, Single
Edge and Double Edge Cracked Finite Plate Subjected to Tension Stress
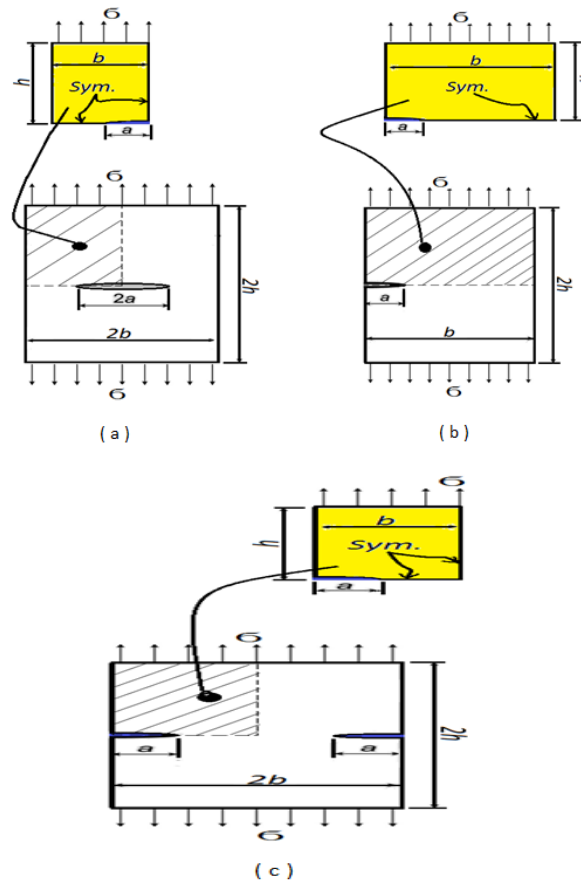
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Figure 2: Cracked Plate Specimens with Dimensions for a) CCT b) SENT C) DENT

Table 1: Theoretical Equations that are Used in this Paper to Calculate Stress Intensity Factors Ki

Cracked Plate Type	Eq. Formula	Eq. No.	Ref. No.	conditions
CCT	$K_I = \sigma\sqrt{\pi a}\left[\sqrt{\sec\left(\frac{\pi a}{2b}\right)}\left[1 - 0.025\left(\frac{a}{b}\right)^2 + 0.06\left(\frac{a}{b}\right)^4\right]\right]$	(1)	[8]	---------
	$K_I = \sigma\sqrt{\pi a}\sqrt{\frac{2b}{\pi a}\tan\left(\frac{\pi a}{2b}\right)}$	(2)	[8]	---------
	$K_I = \sigma\sqrt{\pi a}\left(1 + 0.128\left(\frac{a}{b}\right) - 0.288\left(\frac{a}{b}\right)^2 + 1.525\left(\frac{a}{b}\right)^3\right)$	(3)	[8]	(a/b) ≤ 0.5
	$K_I = \sigma\sqrt{\pi a}\sqrt{\sec\left(\frac{\pi a}{b}\right)}$	(4)	[9]	---------
	$K_I = \sigma * \sqrt{\pi * a} * \left\lvert\frac{1 - 0.5\left(\frac{a}{b}\right) + 0.37\left(\frac{a}{b}\right)^2 - 0.044\left(\frac{a}{b}\right)^3}{\sqrt{1 - \frac{a}{b}}}\right\rvert$	(5)	[8]	---------
SENT	$K_I = \sigma\sqrt{\pi a}\left[1.12 - 0.23\left(\frac{a}{b}\right) + 10.56\left(\frac{a}{b}\right)^2 - 21.74\left(\frac{a}{b}\right)^3 + 30.42\left(\frac{a}{b}\right)^4\right]$.	(6)	[10]	(h/b) $= 2$
	$K_I = \sigma\sqrt{\pi * a}\left[\frac{\sqrt{\frac{2b}{\pi a}*\tan\frac{\pi a}{2b}}}{\cos\frac{\pi a}{2b}}\right]\left[0.752 + 2.02\left(\frac{a}{b}\right) + 0.37\left(1 - \sin\frac{\pi a}{2b}\right)^3\right]$	(7)	[8]	---------

$K_I = \sigma\sqrt{\pi * a}\left[0.265\left(1 - \frac{a}{b}\right)^4 + \frac{0.857 + 0.265\frac{a}{b}}{\left(1 - \frac{a}{b}\right)^{\frac{3}{2}}}\right]$	(8)	[8]	(a/b) < 0.6
$K_I = \sigma\sqrt{a}\left[1.99 - 0.41\left(\frac{a}{b}\right) + 18.7\left(\frac{a}{b}\right)^2 - 38.48\left(\frac{a}{b}\right)^3 + 53.85\left(\frac{a}{b}\right)^4\right]$	(9)	[9]	---------
$K_I = \sigma\sqrt{a}\left[1.12\sqrt{\pi} - 0.41\left(\frac{a}{b}\right) + 18.7\left(\frac{a}{b}\right)^2 - 38.48\left(\frac{a}{b}\right)^3 + 53.85\left(\frac{a}{b}\right)^4\right]$	(10)	[2]	---------

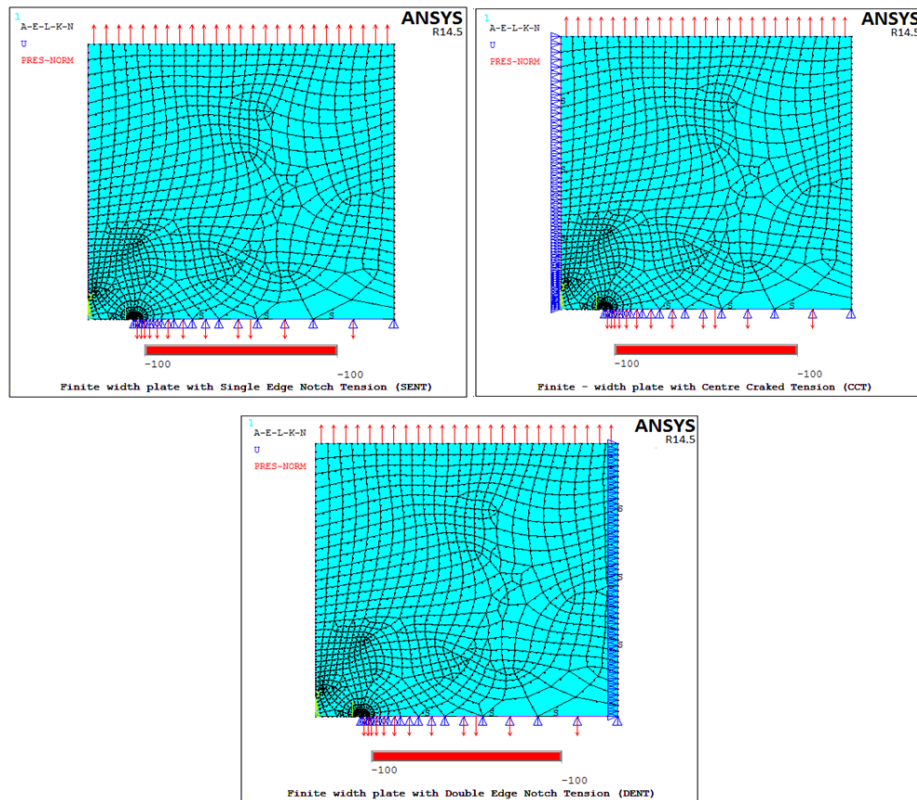|				
DENT	$K_I = \left[1.12 + 0.204\left(\frac{a}{b}\right) - 1.197\left(\frac{a}{b}\right)^2 + 1.93\left(\frac{a}{b}\right)^3\right]\sigma\sqrt{\pi a}$	(11)	[8]	---------
	$K_I = \sigma\sqrt{\pi a}\left[\left(1.122 - 0.561\left(\frac{a}{b}\right) - 0.015\left(\frac{a}{b}\right)^2 + 0.091\left(\frac{a}{b}\right)^3\right)/\left(\sqrt{1 - \left(}\right.\right.$	(12)	[8]	(a/b) < 0.7
	$K_I = \sigma\sqrt{\pi a}\left[\left(1.122 - 0.561\left(\frac{a}{b}\right) - 0.205\left(\frac{a}{b}\right)^2 + 0.471\left(\frac{a}{b}\right)^3\right) - 0.19\left(\frac{a}{b}\right)\right.$	(13)	[8]	---------
	$K_I = \left[1.12 + 0.43\left(\frac{a}{b}\right) - 4.79\left(\frac{a}{b}\right)^2 + 15.46\left(\frac{a}{b}\right)^3\right]\sigma\sqrt{\pi a}$	(14)	[10]	---------
	$K_I = \sigma\sqrt{a}\left[1.12\sqrt{\pi} + 0.76\frac{a}{b} - 8.48\left(\frac{a}{b}\right)^2 + 27.36\left(\frac{a}{b}\right)^3\right]$	(15)	[2]	---------

**Figure 3: Mesh Generation with 556 Elements, 1709 Nodes and
Boundary conditions for a) CCT B) SENT C) DENT**

Comparison between theoretical and Numerical Solutions for Center, Single
Edge and Double Edge Cracked Finite Plate Subjected to Tension Stress
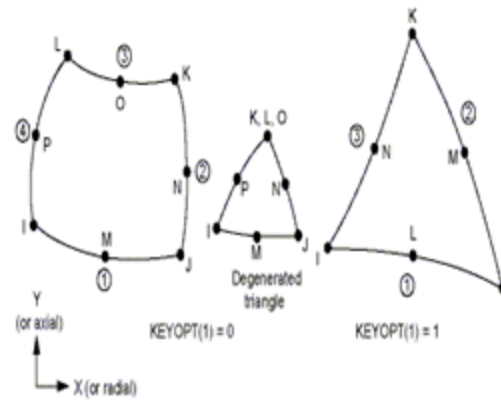
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Figure 4: The Geometry, Node locations, and the Coordinate System
for Element Plane183, ANSYS Help[11]

Table 2: The Cases Studied with the Plate, Solution Types and Number of Figures

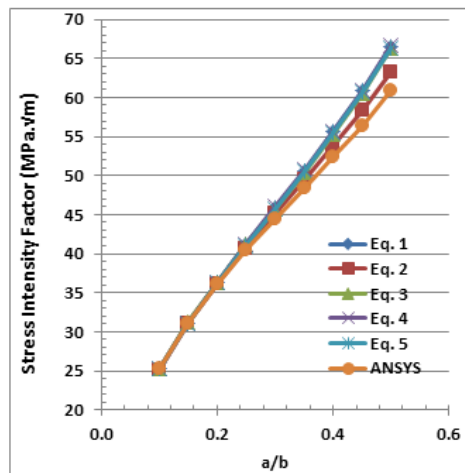| No. of Studied Cases	Changed Parameter in this Case Study		Other Parameters	Numerical & Theoretical Solutions			Numerical Solution Only
				Plate Type & Figure No.			Plate Type & Figure No.
	Name	**Values (m)**		**CCT**	**SENT**	**DENT**	**CCT, SENT& DENT**
I	a/b	0.02	σ_t =100 Mpa b = 0.2 m h = 0.2 m	Figure 5	Figure 6	Figure 7	Figure 14
		0.03					
		0.04					
		0.05					
		0.06					
		0.07					
		0.08					
		0.09					
		0.1					
	Name	**Values (Mpa)**		**CCT**	**SENT**	**DENT**	**CCT, SENT& DENT**
II	σ_t=100 Mpa	50	a = 0.03 m b = 0.2 m h = 0.2 m	Figure 8	Figure 9	Figure 10	Figure 15
		75					
		100					
		125					
		150					
		175					
		200					
		225					
		250					
	Name	**Values (m)**		**CCT**	**SENT**	**DENT**	**CCT, SENT& DENT**
III	h/b	0.02	a = 0.03 m σ_t =100 Mpa b = 0.2 m S	Figure 11	Figure 12	Figure 13	Figure 16
		0.04					
		0.06					
		0.08					
		0.1					
		0.2					
		0.3					
		0.4					

Figure 5: Variation of Stress Intensity Factor with (a/b) Ratio theoretically and Numerically for CCT
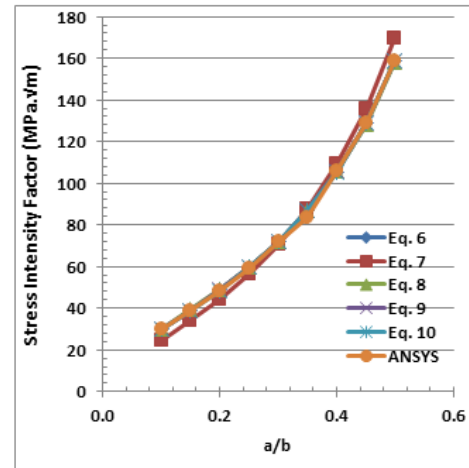
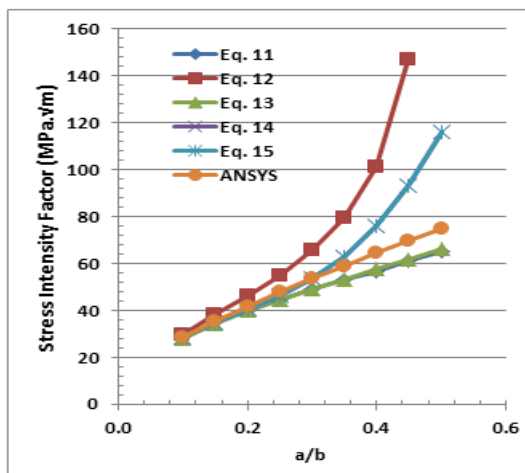**Figure 6: Variation of Stress Intensity Factor with (a/b) Ratio theoretically and numerically for SENT**

Figure 7: Variation of Stress Intensity Factor with (a/b) Ratio Theoretically and Numerically for DENT
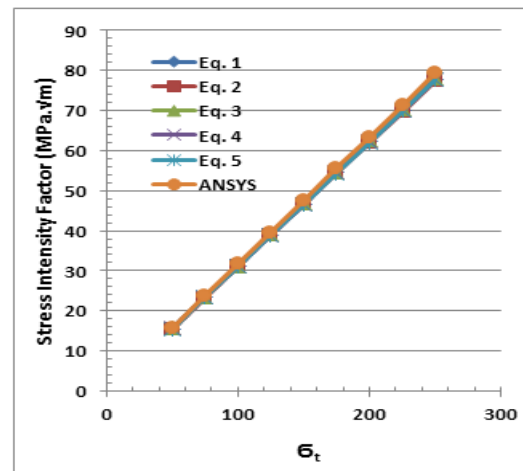
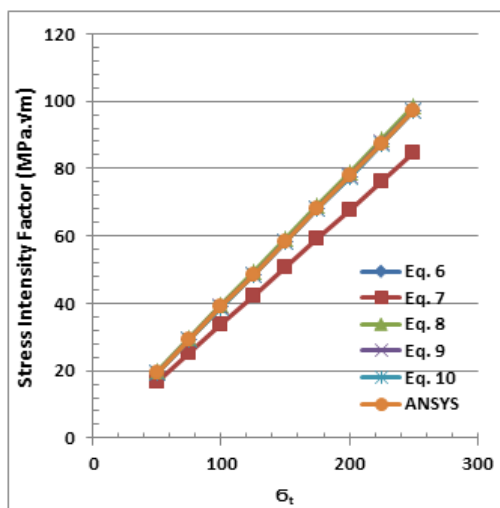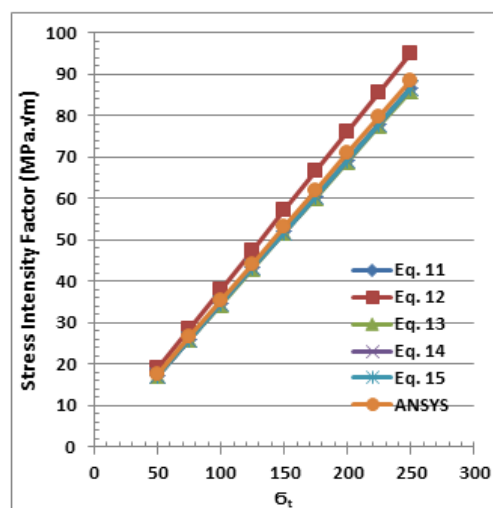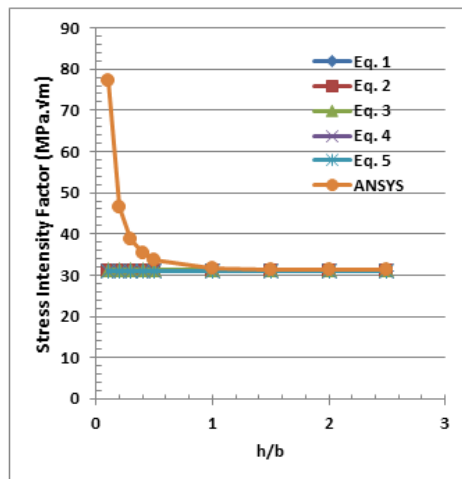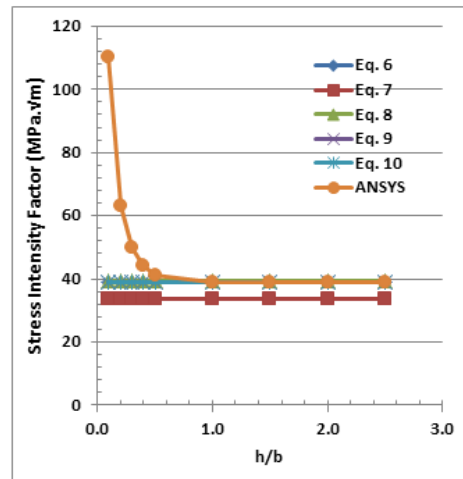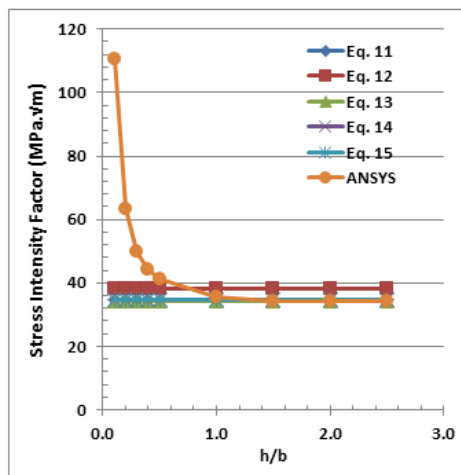**Figure 8 Variation of Stress Intensity Factor With σ_t theoretically and numerically for CCT**

Figure 9: Variation of Stress Intensity Factor with σ_t Theoretically and Numerically for SENT

Figure 10 Variation of Stress Intensity Factor with σ_t Theoretically and Numerically for DENT

Comparison between theoretical and Numerical Solutions for Center, Single
Edge and Double Edge Cracked Finite Plate Subjected to Tension Stress

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**Figure 11: Variation of Stress Intensity Factor with
(h/b) Ratio Theoretically and Numerically for CCT**

**Figure 12 Variation of Stress Intensity Factor with
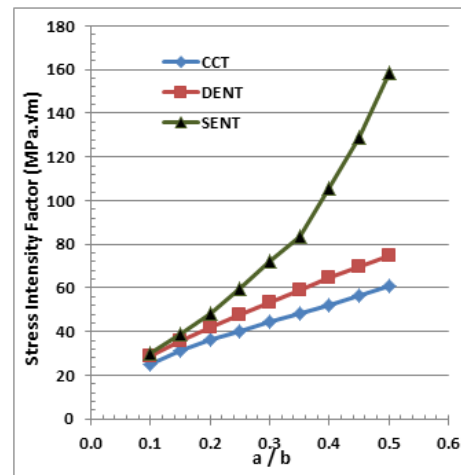(h/b) Ratio Theoretically and Numerically for SENT**

**Figure 13: Variation of Stress Intensity Factor with
(h/b) Ratio Theoretically and Numerically for DENT**

**Figure 14: Variation of Stress Intensity Factor with
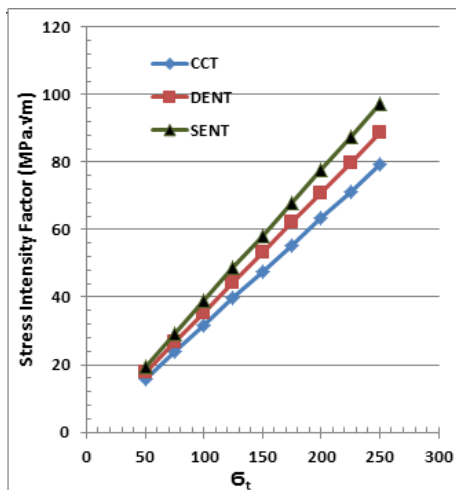(a/b) RATIO Numerically for CCT, SENT and DENT**

**Figure 15: Variation of Stress Intensity Factor with
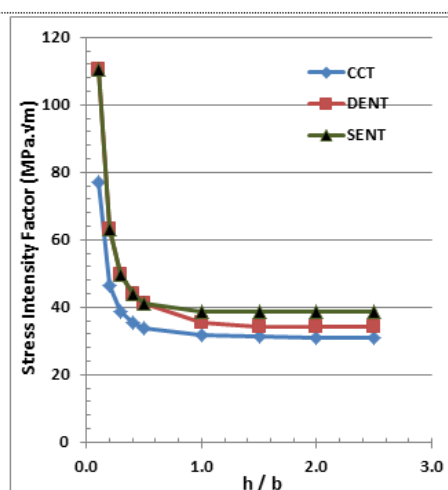Ϭt Numerically for CCT, SENT and DENT**

**Figure 16: Variation of Stress Intensity Factor with
(h/b) ratio Numerically for CCT, SENT and DENT**

(a)

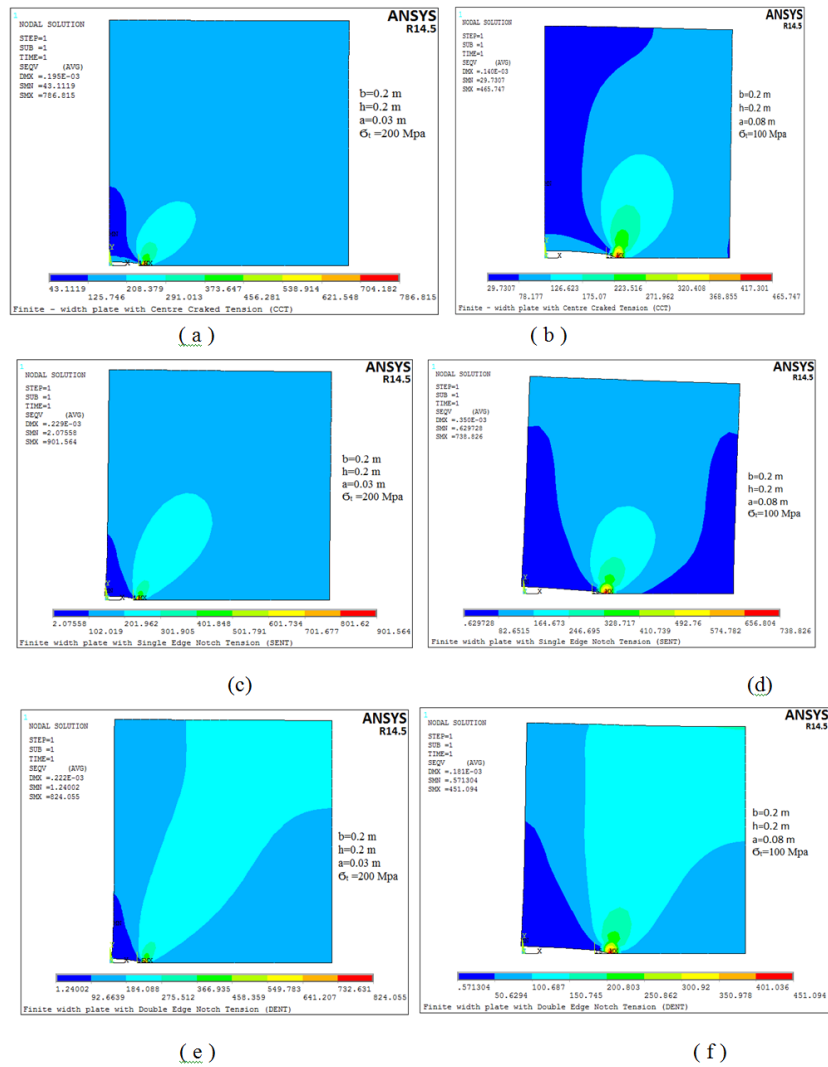(b)

(c)

(d)

(e)

(f)

**Figure 17: Von-Mises Stresses Countor Plots with Deformed Shape for
1) CCT (a & b) 2) SENT (c & d) 3) DENT (e & f)**